



MATHEMATICS

Maximum Marks: 80

Time Allowed: Three Hours

(Candidates are allowed additional 15 minutes for only reading the paper. They must NOT start writing during this time.)

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.

Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B: Internal choice has been provided in one question of two marks and one question of four marks.

Section C: Internal choice has been provided in one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and graph papers are provided.

SECTION A (6 MARKS)

Question -1

(i) If $[A - B]^T = \begin{bmatrix} 1 & 3 \\ \sqrt{3} + 10 & 0 \\ 0 & 4 \end{bmatrix}$ if $A = \begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$

(a) $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

(c) $B = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

(d) None of these

(ii) If $\int x^{-3} \cdot 5^{1/x^2} dx = k \cdot 5^{1/x^2} + C$, then find the value of k.

(a) $k = \frac{1}{2}$

(b) $k = \frac{3}{2}$

(c) $k = \frac{3}{3}$

(d) None of these

(iii) The value of expression $2 \sec^{-1} 2 + \sin^{-1} \left(\frac{1}{2} \right)$ is

(a) $\frac{\pi}{6}$

(b) $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$

(d) 1

(iv) $\frac{dy}{dx} = \frac{x}{y} \left[\frac{x^2 - xy}{x^n + y^n} \right]$ is

(a) $n=3$

(b) $n=5$

(c) $n=2$

(d) $n=4$

(v) If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A'/B')$ is equal to

(a) $\frac{3}{8}$

(b) $\frac{3}{10}$

(c) $\frac{1}{2}$

(d) None of these

(vi) If $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$, find the real value(s) of α .

(a) $\alpha = \frac{1}{5}, -\frac{1}{5}$

(b) $\alpha = \frac{1}{2}, -\frac{1}{5}$

(c) $\alpha = \frac{1}{5}, \frac{1}{8}$

(d) $\alpha = \frac{2}{5}, -\frac{1}{3}$

(vii) If the functions $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{when } x \neq 1 \\ k & \text{when } x = 1 \end{cases}$ is given to be continuous at $x = 1$, then the

value of k is

(a) 1

(b) 2

(c) 3

(d) -1

(viii) The function of $f(x) = \frac{4-x^2}{4x-x^2}$,

- (a) Discontinuous at only one point
- (b) Discontinuous at exactly three Points
- (c) Discontinuous at exactly two Points
- (d) None of these

(ix) Let $f: R \rightarrow R$ be a function defined by $f(x) = [x]$, where $[x]$ denotes greatest integer less than or equal to x . Then, f is

- (a) one-one
- (b) onto
- (c) neither one-one nor onto
- (d) Neither one-one nor onto

(x) Let A and B be two square matrices of same order.

Assertion (A) : In general, $AB \neq BA$.

Reason (R) : $(A + B)^2 = A^2 + 2AB + B^2$.

- (a) Both A and R are correct and R is the correct explanation of A.
- (b) Both A and R are correct but R is not the correct explanation of A.
- (c) A is correct but R is not correct.
- (d) A is not correct but R is correct.

(xi) Let $R = \{(a, a^3) : a \text{ is a prime number less than } 5\}$ be a relation. Find the range of R .

(xii) If the matrix $\begin{bmatrix} -2 & x-y & 5 \\ 1 & 0 & 4 \\ x+y & z & 7 \end{bmatrix}$ is symmetric, find the values of x, y and z .

(xiii) For the set $A = \{1, 2, 3\}$ is defined a relation R in the set A as $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Then, the ordered pair to be added to R to make it the smallest equivalence relation is

- (a) $(3, 1)$
- (b) $(2, 1)$
- (c) $(3, 2)$
- (d) $(2, 3)$

(xiv) A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6 . Then the probability that B is selected...?

(xv) If A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{2}{3}$, then $P(\overline{A} \cap B)$ is equal to

- (a) $\frac{7}{9}$
- (b) $\frac{5}{12}$
- (c) $\frac{1}{9}$
- (d) $\frac{6}{11}$

Question -2

(i) Find $\frac{dy}{dx}$, if $x^3 + y^3 = 3axy$.

OR

(ii) Prove that the function $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on $\mathbb{R}$.

Question -3

Evaluate: $\int_4^5 |x - 5| dx$.

Question -4

Find the point on the curve $\frac{x^2}{9} - \frac{y^2}{16} = 1$ at which the tangent are parallel to x-axis.

Question -5

(i) Find the primitives of the following function $\frac{x-2}{2x^2-11x+5}$.

OR

(ii) Evaluate the integral: $\int \sqrt{x} e^{\sqrt{x}} dx$.

Question -6

Find the domain of the function: $f(x) = \frac{1}{\sqrt{x-|x|}}$

Question -7

If $\sin^{-1} x + \tan^{-1} x = \frac{\pi}{2}$ prove that $2x^2 + 1 = \sqrt{5}$.

Question -8

Evaluate: $\int \frac{x^3}{x^3 + 3x - 3} dx$.

Question -9

(i) The value of a and b, for which $f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$ is differentiable at $x=1$.

OR

(ii) If $y = a \cos(\log x) + \sin(\log x)$, prove that $x^2 y_2 + x y_1 + y = 0$.

Question -10

Three friends toss coins until all three get heads in the same round, then they split the bill. Otherwise, the restaurant pays (just a fun twist).

- (a) Probability in one round that all get heads?
- (b) Probability they get a free meal (never all heads in first 3 rounds)?
- (c) Probability they split the bill exactly on round 5?

OR

Three Teaching Methods

Students taught by:

- Method 1 (40% of students), pass rate 60%
- Method 2 (35% of students), pass rate 80%
- Method 3 (25% of students), pass rate 95%

A randomly selected student passed the exam.

- (a) Define events.
 (b) Find probability they were taught by Method 3.
 (c) Which method is most likely given they passed?

Question -11

Solve the following system of equations, using matrix method:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} + \frac{20}{z} = 2; x, y, z \neq 0.$$

Question -12

- (i) Find the particular solution of the differential equation: $(1 + x^2) \frac{dy}{dx} = (e^{m \tan^{-1} x} - y)$, given that $y = 1$ where $x = 0$.

OR

- (ii) Evaluate : $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$.

Question -13

- (i) A square tank of capacity 250 cubic metres has to be dug out. The cost of land is Rs.50 per sq. m. The cost of digging increases with the depth and for the whole tank it is $400xh^2$ where h metres is the depth of the tank. What should be the dimensions of the tank so that the cost would be minimum?

OR

- (ii) Find the maximum surface area of a circular cylinder that can be inscribed in a sphere of radius R .

Question -14

There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean of X .

Section B 15 MARKS

Question -15

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) If the directions cosines of a line are k, k, k , then
(a) $k > 0$ (b) $0 < k < 1$ (c) $k = 1$ (d) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$
- (ii) The two vectors $\vec{j} + \vec{k}$ and $3\vec{i} - \vec{j} + 4\vec{k}$ represent the two sides AB and AC, respectively of $\triangle ABC$. Find the length of the median through A.
- (iii) Write the value of p for which the vectors $3\vec{i} + 2\vec{j} + 9\vec{k}$ and $\vec{i} - 2\vec{j} + 3\vec{k}$ are parallel vectors.
- (iv) Find the direction cosines of the line: $\frac{x-1}{2} = -y = \frac{z+1}{4}$.
- (v) A plane passes through the points $(2,0,0)$, $(0, 3, 0)$ and $(0, 0, 4)$. Then find the equation of plane.

Question -16

Find the length of the perpendicular from origin to the plane $(3\vec{i} + 4\vec{j} - 12\vec{k}) \cdot \vec{r} + 39 = 0$.

OR

Find value of λ if vectors $\lambda\vec{i} + \vec{j} + 2\vec{k}$, $\vec{i} + \lambda\vec{j} - \vec{k}$ and $2\vec{i} - \vec{j} + \lambda\vec{k}$ are coplanar.

Question -17

Determine the equation of the line passing through the point $(-1, 3, -2)$ and perpendicular to the line: $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

OR

Find image of the point $(3, -2, 1)$ in the plane $3x - y + 4z = 2$.

Question -18

Find the area of the region bounded by $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$

Section C 15 MARKS

Question -19

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) The demand function of a monopolist is given by $x = 100 - 4p$. The quantity at which $MR=0$ will be :
 (a) 25 (b) 10 (c) 50 (d) 30
- (ii) The purpose of regression is to
 (a) maximize the r^2
 (b) explain the mean of the independent variable.
 (c) explain the variation of the dependent variable.
 (d) explain the variation of the independent variable.
- (iii) For the lines of regression $4x + 2y = 4$ and $2x + 3y + 6 = 0$, find the mean of 'x' and the mean of 'y'. What is the revenue function?
- (iv) A company receives a shipment of 500 scooters every 30 days. From experience it is known that the inventory on hand is related to the number of days x . Since the shipment, $I(x) = 500 - 0.03x^2$, the daily holding cost per scooter is Rs.0.3. Determine the total cost for maintaining inventory for 30 days.
- (v) If the revenue function is $R(x) = \frac{2}{x} + x + 1$, then find the marginal revenue function.

Question -20

- (a) The cost function $C(x)$ for x breads is given by $C(x) = \text{Rs.}(3.5x + 1200)$. Each bread is put to a special levy of 20 paise for Odisha cyclone victims. Then, if each bread is sold for Rs. 6 determine the minimum number of breads that should be produced and sold to ensure no loss.
- (b) The average cost of producing x units of commodity is given by the equation $AC = \frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}$. Find the marginal cost function (MC).

Question -21

The marks obtained by 10 candidates in English and Mathematics are given below:

Marks in English	Marks in Mathematics
20	17
13	12
18	23
21	25
11	14
12	08
17	19
14	21
19	22
15	19

Estimate the probable score for Mathematics if the marks obtained in English are 24.

Question -22

- (a) A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items of storage. An electronic sewing machine cost him Rs. 360 and a manually operated sewing machine Rs. 240. He can sell an electronic sewing machine at a profit of Rs. 22 and a manually operated sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Make it as a LPP and solve it graphically.

OR

- (b) An oil company requires 12,000, 20,000 and 15,000 barrels of high-grade, medium grade and low grade oil, respectively. Refinery A produces 100, 300 and 200 barrels per day of high grade, medium grade and low grade, respectively, while refinery B produces 200, 400 and 100 barrels per day of high-grade, medium grade and low grade, respectively. If refinery A costs Rs. 400 per day and refinery B costs Rs. 300 per day to operate, how many days should each be run to minimize costs while satisfying requirements.

